Voltage oriented control of three-phase PWM rectifier using space vector modulation and input output feedback linearization theory

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Abstract—The current energy challenges encourage the researchers to design and optimize the devices allowing the conversion and the control of energy. The power converters consist in adapting the source of energy to the needs for the load. In this paper, we present the study of voltage oriented control with space vector modulation (VOC-SVM) strategies for three-phase PWM rectifiers. This strategy of control founded on the transformation between stationary coordinates α-β and synchronous rotating coordinates d-q, it is based on the input-output feedback linearization controller for which the objective is to control the grid current’s d- and q-axis component, while the dc-bus voltage is maintained at the desired level and satisfy the unity power factor (UPF) operation. The strategy VOC-SVM ensures high static performance and fast transient response by using internal current control loop. Moreover, the studies were carried out by using simulation under the environment Matlab/Simulink.

Keywords—Voltage oriented control; PWM rectifier; Space vector modulation; Input–output feedback linearization.

I. INTRODUCTION

Several mountings of three-phase PWM rectifiers are used in different industrial sectors require significant power. Indeed, these converters are nonlinear loads which absorb non-sinusoidal currents and consume the reactive power which leads to a low power factor and a distortion of the line voltage [1]. Consequently, the three-phase PWM rectifiers are one of the most promising solutions thanks to advantages such as, the consumption of a current close to a sinusoid by reducing its harmonic content, the reactive power control absorbed and the dc-bus voltage and ensured it a bidirectional transfer of power [2].

The control of the PWM rectifier can be considered a common problem with controlling a PWM inverter. Several techniques of control of PWM converters were presented in the literature which aims to have a reduced harmonic distortion ratio and a precise power control [3-4]. They can be classified, according to their principles in two categories: techniques based on the voltage and techniques based on the virtual-flux which were designed in [5] - [9]. The DPC strategies aims to eliminate the pulse width modulation block and the internal control loop by replacing them with a switching table whose inputs are the errors between the reference values and the measured values and were developed in [6-7],[10],[12]. The VOC method is founded on the coordinate’s transformation between the fixed coordinates system α-β and coordinates d-q, where the current control is carried in the voltage space vector of reference frame oriented [2-3], [14].

In this paper, we propose a new approach to control the three-phase PWM rectifiers which is the VOC with space vector modulation (SVM). For this we have implemented a non-linear control technique that takes into account non-linearity of the system, this method is based on an input-output feedback linearization so it ensures, the control of the dc-bus voltage and the currents (id-iq) and also ensure unity power factor.

The paper is structured as follows. Sections II present the mathematical model of the three-phase PWM rectifier which is followed by a description of the VOC-SVM in section III. The section IV will be devoted to the study of nonlinear control input-output feedback linearization. The section V is interested to the simulation results. Finally, conclusions are presented.

II. MATHEMATICAL MODEL

Fig. 1 shows the block diagram of the three-phase PWM rectifier, in which $e_a$, $e_b$ and $e_c$ represent the power source phase voltages; $R$ and $L$ is line resistance and line inductance respectively; $i_a$, $i_b$ and $i_c$ are the line current of PWM rectifier; $v_a$, $v_b$ and $v_c$ are the AC side voltages of the rectifier; $C$ is dc-link capacitor; $i_{dc}$ is dc-link current; $i_L$ is load current; $R_L$ is load resistance.
From Fig. 1, we can deduct the equations of the system in the two-phase synchronous rotation d-q coordinate:

\[
\begin{align*}
\frac{di_d}{dt} &= \frac{-R}{L} i_d + \frac{1}{L} (e_d - v_d) \\
\frac{di_q}{dt} &= \frac{-R}{L} i_q - \frac{1}{L} (e_q - v_q) \\
\frac{dv_{dc}}{dt} &= -v_{dc} + \frac{(e_d i_d + e_q i_q)}{CR_L + C v_{dc}}
\end{align*}
\]

(1)

III. PRINCIPLE OF VOLTAGE ORIENTED CONTROL-SPACE VECTOR MODULATION

A. System configuration

VOC is based on the orientation of the current vector in the same direction as that of the voltage vector, by controlling the current vector in the two rotating coordinates dq. This control strategy it possible to get a decoupled control of the two components of the current vector in the synchronous coordinates directed in the same direction of the voltage vector of the grid. This which ensures the PWM rectifier should draw of the sinusoidal line currents. This structure present a major disadvantage lies in its variable switching frequency. A purpose of obtaining a constant switching frequency, one uses the VOC-SVM. Consequently, the performances of the control strongly depend on the ability of the control loop input-output feedback linearization controller [8-11], [14]. As shown in Fig. 2, the total topology of the voltage oriented control with space vector modulation for a three-phase PWM rectifier, \( S_a, S_b \) and \( S_c \) represent the switching state of the rectifier; \( i_{dq} \) are d-q components of the grid current; \( e_{dq} \) are d-q components of the grid voltage; \( u_{dq} \) are d-q components of the input voltage of the rectifier; \( u_{\alpha\beta} \) are \( \alpha-\beta \) components of the input voltage of the PWM rectifier.

The VOC is based on the synchronization by the PLL (Phase Locked Loop) to estimate and filter the angle of the source and the instantaneous amplitude of the equivalent phase of a three-phase system. The PLL guarantee that it followed by phase of the direct component of the grid voltage,
\( e_\beta \) in order to eliminate the component in squaring, \( e_\alpha \) which produces the estimated phase, \( \theta \), is equal to the phase of the grid [9].

For the transformation of coordinates \( \alpha-\beta \) to the coordinates \( d-q \) the angle of voltage vector is given by:

\[
\sin \theta = \frac{e_\beta}{\sqrt{(e_\alpha)^2 + (e_\beta)^2}} \tag{2}
\]

\[
\cos \theta = \frac{e_\alpha}{\sqrt{(e_\alpha)^2 + (e_\beta)^2}} \tag{3}
\]

**B. Space Vector Modulation**

The SVM treats the control signals directly in two-phase \( (\alpha-\beta) \) coordinate frame. This strategy supposes that one works within the framework of a numerical control and that a control algorithm already determined the desired components \( u_\alpha \) and \( u_\beta \).

For different combinations of control \( (S_a, S_b, S_c) \), the three-phase PWM rectifier generates only eight voltage vectors \( V_i \) \( (i=0, ..., 7) \) in stationary \( \alpha-\beta \) coordinates systems as shown in Fig. 3, of which two are null and six have a module \( \sqrt{(3/2)} \times V_{dc} \). The rectifier can provide in a way instantaneous and exact of the type voltages \( V_i \). We cannot realize a voltage \( (u_\alpha, u_\beta) \) that in average value and one a sampling period \( T_e \). It must therefore apply vectors of achievable voltage for durations longer adequate on this interval \( T_e \).

To minimize the voltage ripples and compensated for the harmonics, it is necessary to realize \( (u_\alpha, u_\beta) \) with the two voltage vectors \( V_i \) which are closest.

In the case where the reference vector is in the sector \( i (i=1, ..., 6) \), then \( t_i, t_{i+1} \) and \( t_0 \) are respectively the times application adjacent vectors \( V_i, V_{i+1} \) and \( V_0, V_7 \). These times are calculated by the following equations:

\[
t_1 = \frac{\sqrt{3}u_\alpha - \sqrt{2}u_\beta}{2V_{dc}} \tag{4}
\]

\[
t_2 = \frac{\sqrt{2}u_\beta T_e}{2V_{dc}} \tag{5}
\]

\[
t_0 = T_e - t_1 - t_2 \tag{6}
\]

**IV. PRINCIPLE OF THE INPUT OUTPUT FEEDBACK LINEARIZATION CONTROLLER**

The input-output feedback linearization control approach allows of mathematically transforming the non-linear system in a partially or completely linear system in order to have used the linear control methods.

Either the system defined by:

\[
\begin{align*}
&\dot{x} = f(x) + g(x)u \\
y = h(x)
\end{align*}
\]

Or:

\[
x = [x_1, x_2, ..., x_p]
\]

\[
u = [u_1, u_2, ..., u_p]
\]

\[
y = [y_1, y_2, ..., y_p]
\]

represents the vector of the output,

\( f(x), g(x) \) and \( h(x) \) are non-linear functions.

This control technique consists in finding a relation linear between the input and the output by deriving the output until at least an entry appears by using the expression:

\[
y^{(r)} = L_f^{(r)} h_j(x) + \sum_{i=1}^{p} L_j^{-1} h_j(x) u_i
\]

\[
\tag{8}
\]

![Fig. 3. Space vector modulation of VOC for three-phase PWM rectifier](image-url)
Or:

\[ L_{qj} L_f^{(r_j-1)} h_j(x) \neq 0 \]  \hspace{1cm} (9)

\( r_j \) is the relative degree of the system.

The derivatives of Lie are defined by the following relationship:

\[ L_f h(x) = \left( \frac{\partial h(x)}{\partial x} \right) f \]  \hspace{1cm} (10)

By iteration, we have the following equation:

\[ L_f^i h_j = L_f (L_f^{i-1} h_j); i = 2,3,... \]  \hspace{1cm} (11)

The matrix form of the expression (8) is given by:

\[ \begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_p' \end{bmatrix} = D(x) + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix} \]  \hspace{1cm} (12)

Or D(x) is the matrix of decoupling of the system.

The control law for linearization and decoupling input-output is given by:

\[ u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}^T = \begin{bmatrix} v_d \\ v_q \end{bmatrix}^T \]  \hspace{1cm} (16)

\[ u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}^T = \begin{bmatrix} i_d \\ i_q \\ v_{dc} \end{bmatrix}^T \]  \hspace{1cm} (17)

The equation (15) is given by the following simplified form:

\[ \mathcal{E} = f(x) + g(x)u \]  \hspace{1cm} (18)

With:

\[ \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} x_1 + \frac{e_d}{L} \\ -\frac{R}{L} x_2 - \frac{e_q}{L} \\ -\frac{x_3}{CR_L} + e_q x_1 + e_{q_d} \end{bmatrix} \] \hspace{1cm} (19)

\[ g = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \] \hspace{1cm} (20)

A. Calculation of the relative degrees

The output variables of the PWM rectifier are given by the following system:

\[ \begin{align*}
    y_1 &= x_1 = i_d \\
    y_2 &= x_2 = v_{dc} \\
    y_2 &= x_3 = v_{dc}
\end{align*} \hspace{1cm} (20)

In order to calculate the relative degree \( r_i \) associated with each output variable \( y_i \) chosen, we derive successively the expression (20) which gives:

\[ \mathcal{E} = f_2 - \left( \frac{1}{L} \right) u_2 \] \hspace{1cm} (21)
The relative degree associated the output variables $y_1$ and $y_2$ are respectively $r_1=1$ and $r_2=2$. This indicates that the total relative degree of the system is $r = 3$. This shows the inexistence of internal dynamics.

B. Linearization of the system

We consider the derivative $y_1$ and $y_2$ of the system (21) and relation (22). We obtain:

$$
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix}
= 
\begin{bmatrix}
a_1 \\
a_2
\end{bmatrix}
+ 
D(x)
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
$$

(23)

With:

$$
\begin{bmatrix}
a_1 \\
a_2
\end{bmatrix}
= 
\begin{bmatrix}
f_1 \\
\frac{f_1}{C_x^2} \\
\frac{f_1}{C_x^2} + \frac{1}{CR_L} + e_x x_1 + e_q x_2
\end{bmatrix}
$$

(24)

And

$$
D(x) = 
\begin{bmatrix}
0 & \frac{1}{L} \\
-\frac{e_d}{LC_x} & -\frac{e_q}{LC_x}
\end{bmatrix}
$$

(25)

This give

$$
\text{det}(D(x)) = \frac{-e_d}{LC_x^3}
$$

(26)

The determinant of the matrix $D(x)$ is different than zero; there by $D(x)$ is an invertible matrix. Consequently, the control law for the linearization of the system is given by:

$$
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
= D^{-1}(x)
\begin{bmatrix}
v_1 - a_1 \\
v_2 - a_2
\end{bmatrix}
$$

(27)

Substituting (27) in (23), we obtain a linear system and decoupled such as:

$$
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
= 
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix}
$$

(28)

The errors of followed trajectories of reference are defined as follows:

$$
\begin{bmatrix}
e_1 = i_{q\text{ref}} - i_q \\
e_2 = v_{dc\text{ref}} - v_{dc}
\end{bmatrix}
$$

(29)

The system (29) makes to determine the dynamic closed loop:

$$
\begin{bmatrix}
\dot{e}_1 + k_1 e_1 = 0 \\
\dot{e}_2 + k_2 \dot{e}_1 + k_3 e_2 = 0
\end{bmatrix}
$$

(30)

So, the expressions of $v_1$ and $v_2$ are given by:

$$
\begin{bmatrix}
v_1 = k_1 e_1 \\
v_2 = k_3 e_2 - k_2 \dot{e}_1
\end{bmatrix}
$$

(31)

With $k_i$ are positive coefficients allowing to imposing the desired dynamics.

VI. RESULTS AND DISCUSSION

To expose the performances of strategy VOC-SVM applied to a model of PWM rectifier, we present in this section the different results of numerical simulation. These results have been statements for a nonlinear control based on input-output linearization of the dc-link voltage and the current. The parameters of the system are defined in table 1.

The simulations are performed using Matlab/Simulink, with a purely sinusoidal supply line voltage and balanced; the simulation study was carried out with two principal objectives:

- to explain and present the stability of operation of the technique suggested,


<table>
<thead>
<tr>
<th>TABLE 1. PARAMETERS OF THE SIMULATED SYSTEM</th>
</tr>
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<tbody>
<tr>
<td>Parameters of circuit</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>Sampling frequency</td>
</tr>
<tr>
<td>Line resistance R</td>
</tr>
<tr>
<td>Grid frequency f</td>
</tr>
<tr>
<td>Line inductance L</td>
</tr>
<tr>
<td>Load resistance R_L</td>
</tr>
<tr>
<td>dc-link voltage V_m</td>
</tr>
<tr>
<td>Grid phase voltage E</td>
</tr>
<tr>
<td>dc-link capacitor C</td>
</tr>
</tbody>
</table>
to expose the dynamic performances of the control voltage.

Various tests of simulation were done to verify performance of the technique VOC-SVM with the linearization input-output controller. Figs. 4, 5 and 6 shows the simulated output waveforms of the PWM rectifier in different conditions at unity power factor (UPF), the plan (α, β) is divided into six sectors.

One can note that the DC voltage follows the reference value and the instantaneous currents \( i_d, i_q \) are controlled with their references, and a line voltage in phase with the line current (UPF).

In the case or load and the reference voltage is constant. In the Fig. 4(a), we observe waveforms the line current and line voltage, in Fig. 4(b), the dc-link voltage is constant and follows the reference value. The three-phase sinusoidal grid currents waveforms is show in Fig. 4(c), the \( i_q \) current waveform contains few harmonic distortion, which gives a low total harmonic distortion (THD) equal to 2.7%. The line currents in (d-q) coordinate frame are illustrated in Fig. 4(d).

Fig. 5. presents the various waveforms of the system for a step of \( V_{dc} \) reference which varies between 600V and 700V (b), and we can see the dynamic behavior of the proposed technique such as the good control of the instantaneous currents \( i_d, i_q \) (d). The line current (c) responds well to the imposed variation of \( V_{dc,ref} \), it is quickly established after a phase of transition.

Fig. 6 is interested in the effect of the step change of the load between 100Ω and 50Ω on the operation of system under the UPF. In this case the load is purely resistive, its reduction causes the increase in the current \( i_a \) (a). This causes the

---

Fig. 4. Simulation results of input-output feedback linearization VOC-SVM of PWM rectifier, \( V_{dc,ref} = 600 \text{ V}, i_{q,ref} = 0 \text{ A}, \text{THD} = 2.7\% \);
(a) phase grid voltage and grid currents at UPF; (b) dc-link voltage waveform;
(c) three-phase grid currents; (d) line currents in (d-q) coordinate frame.

Fig. 5. Simulation results of input-output feedback linearization VOC-SVM of PWM rectifier at step change of the \( V_{dc,ref} \) (600V to 700V);
(a) phase grid voltage and grid currents; (b) dc-link voltage waveform;
(c) three-phase grid currents; (d) line currents in (d-q) coordinate frame.
variation of the current $i_d$ (d), this last increases with the decrease of the load, without affecting the current $i_q$ (UPF).

VII. CONCLUSION

This work proposes the modeling and simulation of voltage oriented control space vector modulation based on nonlinear control of PWM rectifier. The mathematical models of the global system are presented. The simulation results showed that this proposed strategy and proven their effectiveness into maintain the dc-bus voltage and the current at the desired level, as well as harmonics reduction. The proposed technique gives good static, dynamic performances via internal current control loop and has an excellent robustness.

REFERENCES


